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Statistical modeling of changes in relative sea level in Maine during the Holocene Era†

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Understanding past relative sea-level changes is important to a number of social and scientific questions, including the effects of global climate change and future land-use planning under scenarios of accelerated sea-level rise with a concomitant increased threat to coastal areas around the world. In particular, accurately characterizing millennial sea-level changes is important in evaluating vertical movements of the Earth's crust that happen in response to the advances and retreats of ice sheets during long-term climatic cycles. In this paper, we analyze sea-level data from several Maine salt marshes previously reported in a paper from the geological literature. We address these data and questions of geological interest with a 'smooth transition' model. Copyright © 2013 John Wiley & Sons, Ltd.

Keywords: bootstrap; carbon dating; geology; measurement error; segmented regression; smooth transition

1. INTRODUCTION

From stories of lost cites to migrations across now-sunken land bridges to questions of the effects of global warming, there has long been both public and scientific interest in understanding past changes in the height of the sea relative to the height of land (relative sea level or RSL). At present, understanding the past history of sea level is critical for predicting the effect of natural process and human-induced climate change on future sea-level rise and shoreline retreat. In this paper, we analyze 6000 years of RSL data collected from four sites along the Maine coast (USA). We address these data with a parametric 'smooth transition' model and address questions of geological interest.

On time scales of less than millions of years, several processes act to change RSL, such as the following:

1. global climate-induced changes in the total volume of water in the ocean due to melting of glaciers or the thermal expansion of water,
2. slow vertical movements of the continents associated with depression and subsequent rebound of part of the Earth's crust due to the expansion and contraction of continental ice sheets during the last ice age, and
3. local and relatively sudden uplift or subsidence of coastal regions associated with earthquakes, other tectonic activity, or sediment compaction.

Evidence of these processes consists both of historical records of water level from tide gauges maintained for navigational purposes and of geologic evidence for higher and lower RSLs at times from hundreds to hundreds of thousands of years ago. Such geologic evidence, examples of which are the subject of this paper, consists of identifying either marine deposits exposed above present sea level or terrestrial deposits now found below sea level and determining the age of these deposits. During the time period relevant to this study (100–6000 years before present), age is determined by radiocarbon dating, which relies on measuring the radioactive decay of organic carbon contained in the deposits. Thus, geologic measurements of past RSL result in a series of data points (age versus past mean tide level minus present mean tide level) that describe the RSL history at a given place. The problem facing histories of RSL through time (e.g., Pirazzoli, 1997).

A large number of modern geologic studies of sea level are concerned with the first and second aforementioned processes that operate on a time scale of thousands to tens of thousands of years and a spatial scale that includes the whole earth (e. g. Peltier, 1998). These studies

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combine RSL histories from around the globe and fit dynamic models derived from first-principle physical considerations. Over the course of a few thousand years, these models conclude that RSL is approximately linear in time. Although this work develops a self-consistent and physically correct method of dealing with sea-level data in which the RSL at any given point is coupled to RSL at every other point on the globe, it has limited use for interpreting localized variations in RSL histories (aforementioned process 3), which might be caused by earthquake-related deformation, volcanic activity, sediment compaction, or changes in local tidal range (Plag *et al.*, 1998).

At present, however, there is no satisfactory analytical framework available to analyze these types of localized variations. The two most common methods previously used by geologists to summarize localized RSL data are interpolation and heuristic smooths of the data. 'Error envelopes' created by connecting the corners of + / - one standard error bars centered around each point are often displayed. Numerical summaries consisting of confidence intervals for the slopes of segments of the curves that appear to be linear are sometimes included. Comparison of curve estimates (across sites or across researchers within sites) typically rely on either visual identification of seemingly common features or on comparisons of confidence intervals around the estimated slopes. In this paper, we suggest that statistical analysis using parametric non-linear models provides a useful framework for analysis of localized millenial-scale RSL data. This methodology allows objective comparisons of RSL data collected from different sites or by different researchers. We apply this methodology to a set of RSL data from the coast of Maine, USA and show that our procedure yields new geologic insights that were not obvious in previous analyses.

1.1. Gehrels, Belknap, and Kelley (1996)

Like most of the northern half of North America during the last glaciation, the coastline of Maine was covered by the Laurentide Ice Sheet; it became ice-free between 16,000 and 12,000 years ago. During glaciation, the weight of the ice pushed down the land surface in this region. By 15,600 years ago, when the ice sheet began to retreat, the land was still depressed by as much as 170 m; thus, RSL was higher than present, and the coastline was well inland of its present position. Just south of the Laurentide Ice Sheet, an elevated ridge had formed in the Earth's crust, known as the glacial 'forebulge'. After the weight of the ice sheet was removed (16,000 and 12,000 years ago), Maine rebounded upwards, and RSL fell. In the area just south of the ice sheet, land level sank, and RSL rose as a result of the shrinkage of the 'forebulge'. As time went on, the ice sheet retreated northward, and the zone of coastal subsidence moved northwards with the ice. In the last few thousands of years, therefore, the coastline of Maine has been sinking, and RSL is rising.

Work by Gehrels *et al.* (1996) studies the RSL history over the past 6000 radiocarbon (^{14}C) years at four sites along the Maine (USA) coast: Wells, Phippsburg, Gouldsboro, and Machiasport. Their paper creates four site-specific scatterplots of estimated sea level (relative to present) over time in the past 6000 ^{14}C years (the middle and late Holocene period). The ^{14}C time scale deviates somewhat nonlinearly from the calendar time scale, because of variations in the atmospheric concentration of the radioactive ^{14}C isotope over time, but to date, organic samples younger than 50,000 calendar years is conventionally used in geology and archeology. Five thousand ^{14}C years is approximately 5700 calendar years and zero calendar years is equal to zero ^{14}C years.

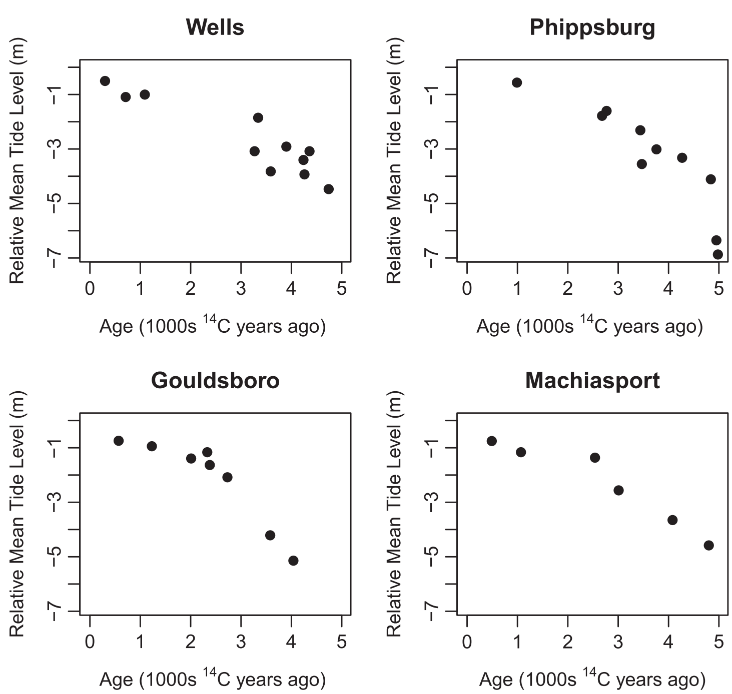

Figure 1. Scatterplots of relative sea level versus time. Sea levels have been rising over time: relative sea level is calculated as previous mean tide minus the mean tide level today. Age is measured in Carbon-14 (^{14}C) years, a scale that is commonly used in geology for samples younger than 50,000 calendar years. Five thousand ^{14}C years is approximately 5700 calendar years, and zero years agree on both scales. Total sample size $(n) = 35$.

Each point in Figure 1 represents a subsample from a core sample taken from the relatively undisturbed area near the bottom of one of the salt marshes. Twenty-seven core samples were taken, twenty-two of them provided one data point, two yielded two data points, two provided three, and one provided four. When multiple data points came from a single core, they were from widely separated points in the core. Where to take core samples and which parts of the sample to analyze were chosen to make it likely that the data would cover the time period of interest. After the subsamples were chosen, each was analyzed to yield estimates of two quantities: deposition date (the x-axis of the scatterplot) and sea level at deposition time (the y-axis). The isotope and chemical methods that are used to produce those estimates are explained in detail in Gehrels *et al.* (1996). We summarize them here briefly.

Deposition time was estimated by ^{14}C dating the organic matter in the subsample and assuming that the age of the organic matter coincides with deposition time. Sea level height is estimated from fossilized *foraminifera* (single-celled shelled organisms) in the subsample. The geologists inferred the mean high-tide level at the time the *foraminifera* were alive from the modern distribution of these organisms on the surface of the salt marshes. This estimates the relative mean tide level at the time the subsample was deposited when it is combined with two other estimated quantities: the depth of the subsample relative to current high-tide level and an estimate of the tidal range when the subsample was deposited.

One focus of Gehrels *et al.* (1996) was to address the question of whether 'differential crustal motion' along the coast of Maine has occurred during the past 5000 years. Simply put, this question asks whether RSL rise along the Maine coast has varied from one locality to the next because of different rates of vertical land movement. If it has, the coast of Maine is geologically less stable than if it has not. In the following sections, we formulate and fit a parametric model to these data and address the aforementioned question with the parameters in the model.

2. A STATISTICAL MODEL FOR SEA LEVEL CURVE ESTIMATES

The next three sections describe our model for the RSL data. Sections 2.1 and 2.2 develop the regression model relating the subsample age and estimated sea level. Section 3 describes how we fit the model and make inferences. The results of estimation, inference, geological interpretations, and the relevance to current sea levels are discussed in Section 4.

2.1. The data and measurement error

Let $i = w, p, g, m$ represent the sites (Wells, Phippsburg, Gouldsboro, and Machiasport) and $j = 1, \ldots, n_i$ represent the observations from a given site. Letting the first index denote the site and the second the repetition, we define the vectors **Y** (RSLs) and **X** (ages) as follows:

$$\mathbf{Y} = \left[Y_{w1}, \ldots, Y_{wn_w}, Y_{p1}, \ldots, Y_{pn_p}, Y_{g1}, \ldots, Y_{gn_g}, Y_{m1}, \ldots, Y_{mn_m}\right]^{\mathrm{T}}, \text{ and}$$

$$\mathbf{X} = \left[X_{w1}, \ldots, X_{wn_w}, X_{p1}, \ldots, X_{pn_p}, X_{31}, \ldots, X_{gn_g}, X_{m1}, \ldots, X_{mn_m}\right]^{\mathrm{T}}.$$

Let $n = n_w + n_p + n_g + n_m$ be the total number of data points. Figure 1 shows the observed data.

Because the age of each subsample is estimated by radiocarbon dating, the observed age is the true age plus error. The radiocarbon dating lab supplies an estimate of the standard deviation of each estimated age ($\hat{\sigma}_{age,ij}$). In this dataset, we find the measurement error to be quite small relative to the range of ages considered. More specifically, let σ^2_{xi} be the variance of the true ages considered at site i. A measure of the amount of measurement error in a data point is the reliability ratio: $\lambda_{ij} = \frac{\sigma^2_{xi}}{\sigma^2_{xi}+\sigma^2_{age,ij}}$ (e.g., Carroll *et al.*, 1995). For these data, the estimated reliabilities are greater than 96% for all ij. As a result, we treat the ages, X_{ij} as if they contain no measurement error. The results of an analysis that included a functional model for the measurement error did not differ substantively from the results reported here (unpublished technical report, Staudenmayer, Altman, Balco, Gehrels, Qiu, and Sullivan).

2.2. Regression model for relative sea level

As discussed in Section 1, global scale models for millennial scale sea level changes exist, but those models do not consider local movements of the earth's crust. Because the purpose of Gehrels *et al.* (1996) is to assess the RSL history on a local level, we develop an empirical model to summarize the data at hand.

The models we use are a series of nested regression models with independent Gaussian errors. Although a few of the cores provided more than one data point (Section 1.1), we did not find evidence for correlation in the error in that small sample. As mentioned in our introduction, global scale sea-level models derived from first principle considerations are approximately linear over the time scale of our data. To allow for local and relatively sudden changes in sea level, we use a slightly more general model. The need for a more general model was suggested by the figures in Gehrels *et al.* (1996).

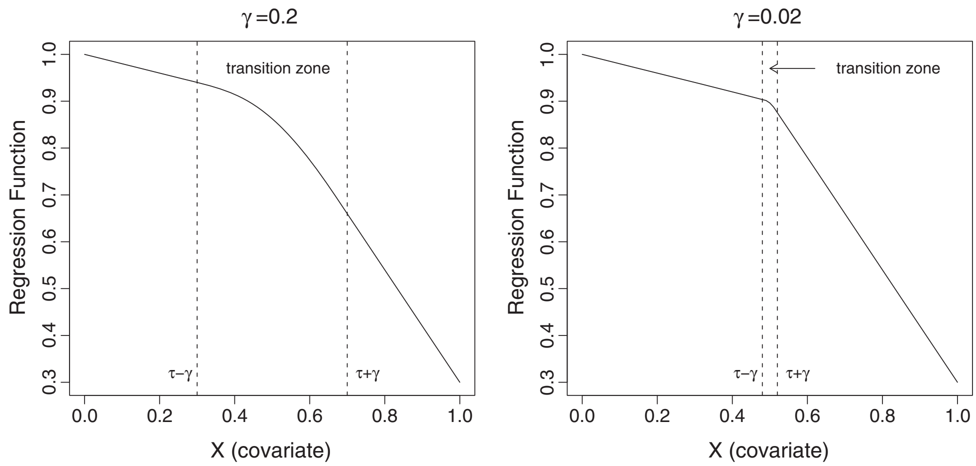

Figure 2. Two illustrations of equation (1) and the smooth transition function with $\beta_0 = 1.0$, $\beta_1 = -0.2$, $\beta_2 = -1.0$, $\tau = 0.5$, and $\gamma = 0.2$ and 0.02. The transition zones, $\tau \pm \gamma$, are shown

The regression model for the conditional mean we use is based on the smooth transition model (e.g., Tishler and Zang (1981) and reviewed in Seber and Wild, 1989, Section 9.4.2a):

$$Y_{ij}|X_{ij} \overset{\text{ind.}}{\sim} N\left\{E(Y_{ij}|X_{ij}), \sigma_i^2\right\} \text{ where}$$

$$E(Y_{ij}|X_{ij}) = \beta_{0i} + \beta_{1i}X_{ij} + \beta_{2i}u(X_{ij} - \tau_i, \gamma),\ j = 1, \ldots, n_i, i = w, p, g, m, \text{ with} \tag{1}$$

$$u(z, \gamma) = 0 \text{ when } z \leqslant -\gamma,$$

$$= \frac{-z^4}{16\gamma^3} + \frac{3z^2}{8\gamma} + \frac{z}{2} + \frac{3\gamma}{16} \text{ when } -\gamma < z \leqslant \gamma, \text{ and}$$

$$= z, \text{ when } z \geqslant \gamma$$

The unknown parameters for sites $i = w, p, g, m$ are: β_{0i}, the intercept; β_{1i}, slope prior to the change area; β_{2i}, the additional slope following the transition interval; τ_i, the center of the transition zone; γ, the half length of the transition zone; and σ_i, the independent Gaussian error standard deviation. When X_{ij} is outside of $\tau_i \pm \gamma$, the model is a segmented linear regression, and as $\gamma \to 0$, the regression function becomes arbitrarily close to a segmented linear model. For positive γ, the mean function has continuous second derivatives in the parameters. See Zang (1980) for more details and Figure 2 for two illustrations of this 'smooth transition' function. Because there is a small amount of data, we constrain γ to be the same over all the sites. We estimated γ from the data, and the estimated γ was less than half the smallest gap between two ages. As a result, the fitted model reverted to a segmented model that has a likelihood that is smooth in its parameters. We note that this mean function is not specific to the geological application; it is simply a smooth function that has segmented regression as a special case. With more data, we believe a non-parametric regression approach would be appropriate, but that is beyond the scope of the current dataset.

The aforementioned model is termed saturated because each site has its own parameters. Nested within this model are simpler models where certain parameters are constrained to be the same across certain sites. In the next subsection, we fit a series of these nested models to the data and arrive at a parsimonious model.

3. FITTING THE MODEL AND INFERENCE

Because $u()$ has continuous second derivatives, fitting the model relies on standard methods. We fit the model by maximum likelihood using the `gnls()` set of tools in the `nlme()` library in R (Pinheiro and Bates, 2000). The data and the R program we used are available from the journal. Nested within the saturated model (1) where all parameters are site specific, we considered a number of variants (suggested by the data) where certain parameters are constrained to be constant across at least some of the sites. Table 1 summarizes a representative few. In those tables, we use the site initials as subscripts when sites share parameters. For instance, $\hat{\beta}_{0wgm}$ means that Wells, Gouldsboro, and Machiasport have the same intercept. We choose model 5 in the table below because it minimizes Bayesian Information Criteria (BIC). There are 12^5 nested models in this family (12 ways to make each of 5 parameters somewhat site specific), and we did not fit all of them. As a result, we cannot be certain that model 5 has the smallest BIC among all the possible nested models. Nonetheless, Figure 3 and Table 1 suggest that model 5 strikes a balance between model parsimony and fidelity to the data.

Table 1. This table summarizes a sample of the models we fit: which parameters are site specific, the number of parameters in the model, and the associated log likelihood and Bayesian Information Criteria. Based on this summary, we choose model 5

Model	Site specific parameters	Number parameter	Log(L)	BIC
1.	None	5	-34.11	85.99
2.	σ_i^2 for each site	8	-28.48	85.41
3.	σ_i^2 for each site and β_{2i}s for each site	11	-13.27	65.65
4.	σ_i^2 for each site and β_{2i}s for each site and τ_p for Phippsburg	12	-11.68	69.58
5.	σ_i^2 for each site and β_{2g} for Gouldsboro and all parameters for Phippsburg	13	-5.25	56.72
6.	all parameters for all sites	20	-3.90	78.91

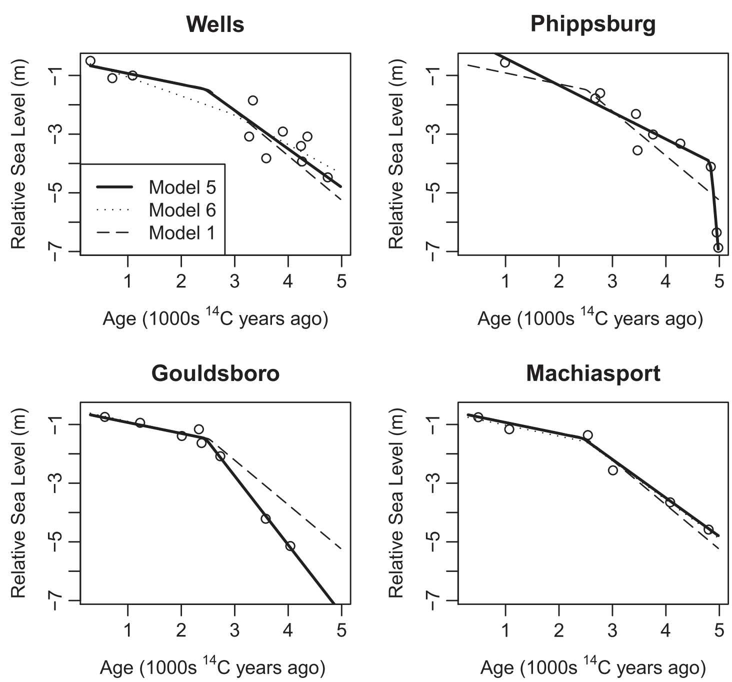

Figure 3. The fitted regression mean functions that correspond to models 1, 5, and 6 are shown with the site-by-site scatterplot data. Note that the fits that correspond to models 5 and 6 (the selected model and saturated model) are nearly identical, but model 5 uses seven fewer parameters, and both of those models have site-specific variances

Because the sample size is small ($n = 35$) relative to the number of parameters in the model, we assess the variability around our estimates by using BC_A bootstrap confidence intervals. The BC_A method of making confidence intervals both reduces bias and accounts for non-normally distributed sampling distributions and often produces intervals with superior properties. A technical description of this method is outside the scope of this paper, but an accessible introduction is in Chapter 14 of Efron and Tibshirani (1993).

We computed the bootstrap by resampling standardized residuals from the estimate of model 5. We chose this way of implementing the bootstrap rather than a method where ($\mathbf{X}$ =Age and $\mathbf{Y}$ =RSL) pairs were resampled (within site) because the geologists selected samples from locations within the marsh with varying depths to ensure that the ages are fully (or appropriately) distributed across the time range of interest. By the principle of superposition, deeper samples are older. As a result, a bootstrap sample with X's that were not distributed over the age range being studied would not accurately represent the way the experiment was conducted.

Table 2 contains estimates of the regression parameters from model 5 and BC_A confidence intervals based on 5000 bootstrap samples. Figure 4 contains curve estimates from model 5 and approximate 95% pointwise confidence envelopes. Each point on the envelope was computed by using the bootstrap sample and the BC_A method.

Table 2. This table contains the estimated regression parameters in model 5 and 95% confidence intervals based on the BC_A method and 5000 bootstrap samples. See Figure 6 also

Parameter	Site(s)	95% Lower	Estimate	95% Upper
$\hat{\beta}_{0wgm}$	Wells Gouldsboro Machiasport	-0.9108	-0.5604	-0.1129
$\hat{\beta}_{0p}$	Phippsburg	-0.1877	0.5113	1.3958
$\hat{\beta}_{1wgm}$	Wells Gouldsboro Machiasport	-0.6126	-0.3722	-0.1040
$\hat{\beta}_{1p}$	Phippsburg	-1.1473	-0.9218	-0.6757
$\hat{\beta}_{2wm}$	Wells Machiasport	-1.3260	-0.9392	-0.6311
$\hat{\beta}_{2p}$	Phippsburg	-49.5381	-18.9675	-12.2023
$\hat{\beta}_{2g}$	Gouldsboro	-2.7803	-1.9736	-1.7014
$\hat{\tau}_{wgm}$	Wells Gouldsboro Machiasport	2.1375	2.4523	2.7212
$\hat{\tau}_p$	Phippsburg	4.7667	4.8311	4.9180

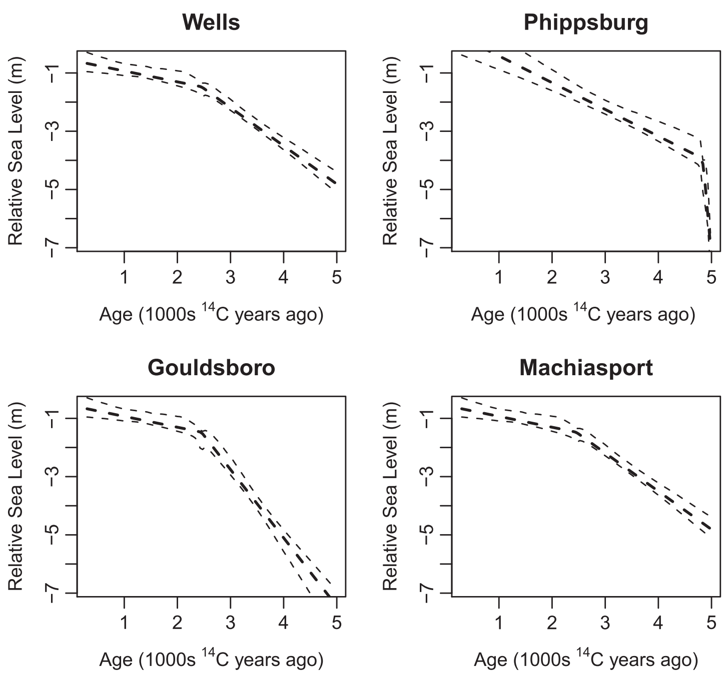

Figure 4. The fitted regression functions from model 5 and BC_A bootstrap-based pointwise 95% envelopes

4. DISCUSSION OF INFERENCES AND GEOLOGICAL SIGNIFICANCE

Figure 5 shows parameter estimates for the selected model (number 5) with the sites arrayed from SW to NE. This figure summarizes the geologic variability of RSL curves along the Maine coast. These parameter estimates lead to three important conclusions.

First, the modeled RSL curve at Phippsburg is significantly different from those at other sites: sea-level rise slowed from middle Holocene rates earlier at this site (larger $\hat{\tau}_p$ which means older change point) but was more rapid than at others thereafter (more negative $\hat{\beta}_{1p}$, which means steeper slope in years since the change point). This has several possible explanations. First, the salt marsh in Phippsburg from which the samples were obtained is located behind a barrier beach and is connected to the open Gulf of Maine by a tidal inlet. As a result, the behavior of the tides at this location and the distribution of the *foraminifera* from which the historical sea levels were inferred are possibly

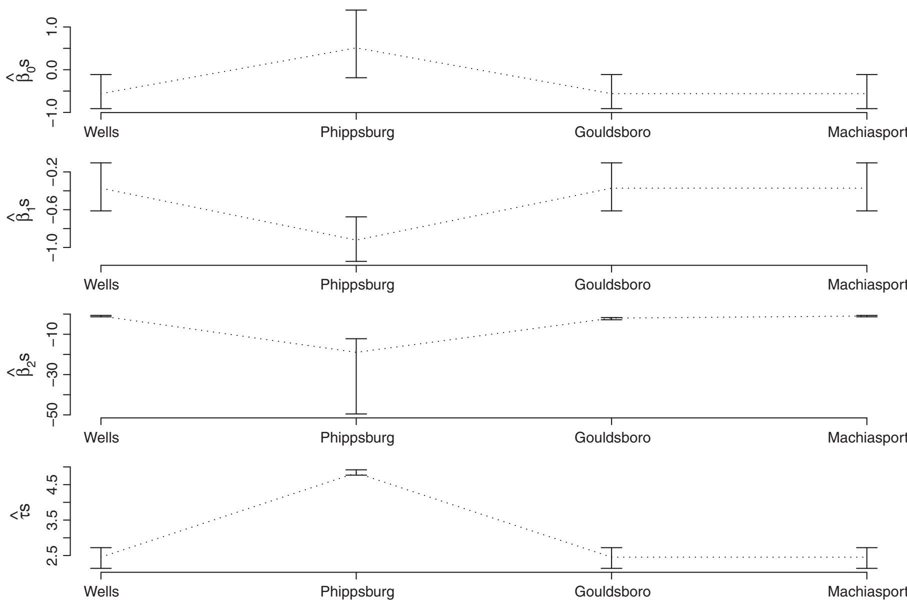

Figure 5. The regression parameter estimates (model 5) and BC_A bootstrap-based 95% confidence intervals are plotted for each site with the sites arrayed from SW to NE

atypical. Second, it is possible that this portion of the Maine coast is unstable relative to surrounding areas. Finally, of course it is possible that there has been an unrecognized data collection error. In any case, further examination of this site is warranted. It would perhaps be rash to base an important conclusion on so few data points. Specifically, the conclusions that the Phippsburg site is different relies on the two oldest samples from that site. Note, however, that these samples came from different core samples and represent a significant fraction of the data collected from that site. Also note that our observation that Phippsburg has an anomalous RSL history differs from the conclusion of Gehrels *et al.* (1996). This points out the need for a consistent and statistically justifiable method of comparing radiocarbon-based RSL histories.

Second, with the exception of anomalous results at Phippsburg, estimated model parameters change slowly with geographic position (Figure 5). The rate of mid-Holocene RSL rise ($\hat{\beta}_{2g}$) is smaller in the Gouldsboro site, but these sites have experienced essentially the same RSL history in the past, approximately 3000 years. These sea level histories are all generally consistent with the histories predicted by the global geophysical models discussed in Section 1 (Peltier, 1998). This observation strongly supports the conclusion of Gehrels *et al.* (1996) that the region of eastern Maine (near Machiasport), where an anomalous uplift in the land was initially suspected, has, in fact, been stable relative to areas further south and west.

Except Phippsburg's, all the estimated intercepts ($\hat{\beta}_{0wgm}$) are negative. Although it is tempting to constrain the regressions to go through the origin because current RSL is ,by definition, zero, this would not be appropriate because our model applies only to the time scale range of the observed data, which does not include the present. Constraining the regressions to go through zero (without adding another change point) would be similar to adding a leverage point to each site's data set. It is said that the negative intercepts indicate that an extrapolation of the model to the present is consistent with an increase in the rate of RSL rise in very recent years. This is also supported by other data such as recent tide gauge records and by more detailed analyses of very recent salt marsh deposits. It is important that our model allows us to correctly infer this fact from much sparser and older data points. Because estimating the changing rate of sea level rise is important to formulating models for the future effects of global warming, it is encouraging that our model may allow us to search for this event even at sites where only a sparse RSL record exists.

The model we present, therefore, provides a significant improvement over previous means of comparing local RSL curves, and earlier, we show that it generated useful geologic insights from relatively sparse data. The basic procedure is flexible enough to accommodate sea-level curves of varying forms and is applicable to a variety of geologic problems concerned with local variations in RSL. However, although the model we use for Gehrels *et al.* (1996)'s data is both flexible enough to fit these data well and parsimonious enough to provide reasonable inferential power, other models certainly might be more appropriate for other data sets. In other settings, a spatial model that includes information about the relative proximity of the marshes might be appropriate. To reliably detect such a structure, one would need more than four marshes though.

Finally, our model can also be used in a power analysis to plan future experiments and determine the required sample size at each site as follows: given (1) a hypothesized relative sea-level curve; (2) roughly feasible ranges for the distributional parameters described in Section 2; and (3) a proposed sample size, one could easily simulate the data one might expect to collect and fit a model to the simulated data. Monte Carlo methods could then be used to estimate the probability of detecting a specific hypothesized difference of a certain size between two sites. If those probabilities are deemed too low, this procedure can be repeated with higher proposed sample sizes until the probabilities were deemed to be large enough.

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REFERENCES

Carroll RJ, Ruppert D, Stefanski LA. 1995. *Measurement Error in Nonlinear Models*. Chapman and Hall: London.

Efron B, Tibshirani RJ. 1993. *An Introduction to the Bootstrap*. Chapman and Hall: London.

Gehrels WR, Belknap DF, Kelley JT. 1996. Integrated high-precision analyses of Holocene relative sea-level changes: lessons from the coast of Maine. *Geological Society of America Bulletin* **108**: 1073–1088.

Pinheiro JC, Bates DM. 2000. *Mixed-Effects Models in S and S-PLUS*. Springer: New York.

Peltier WR. 1998. Postglacial variations in the level of the sea: implications for climate dynamics and solid-earth geophysics. *Reviews of Geophysics* **36**: 603–689.

Pirazzoli PA. 1997. *Sea-Level Changes: The last 20,000 years (Coastal Morphology and Research)*. John Wiley and Sons: New York.

Plag HP, Engen B, Clark TA, Degnan JJ, Richter B. 1998. Postglacial rebound and present day three-dimensional deformations. *Journal of Geodynamics* **25**: 263–301.

Seber GAF, Wild CJ. 1989. *Nonlinear Regression*. John Wiley and Sons: New York.

Tishler A, Zang I. 1981. A new maximum likelihood algorithm for piecewise regression. *Journal of the American Statistical Association* **76**: 980–987.

Zang I. 1980. A smoothing-out technique for min–max optimization. *Mathematical Programming* **19**: 61–77l.

