Forward models of geomorphic processes used to interpret complex cosmogenic-nuclide data sets
Basic idea of exposure dating

Be-10 concentration reflects residence in production zone. Either age or erosion rate.

\[ N = \frac{P}{\lambda + \varepsilon/\Lambda} \left[ 1 - e^{-t_{exp}(\lambda+\varepsilon/\Lambda)} \right] \]

\( P = 10 \)

\( P = 0.05 \)
Basic idea of exposure dating

Be-10 concentration reflects residence in production zone. Either age or erosion rate.

\[ N = \frac{P}{\lambda + \epsilon/\Lambda} \left[ 1 - e^{-t_{exp}(\lambda+\epsilon/\Lambda)} \right] \]

- 42° N - 1200 masl
- 234000 atoms/g \(^{10}\)Be
In most applications, we assert infinite $t$, or zero $E$, based on geomorphic observations.

$$N = \frac{P}{\lambda + \frac{\epsilon}{\Lambda}} \left[ 1 - e^{-t_{exp}(\lambda+\frac{\epsilon}{\Lambda})} \right]$$

Steadily eroding landscapes: eroding long enough to come to production-erosion equilibrium. 0.009 g/cm²/yr = 35 m/Myr.

Glacially transported boulders: no inheritance, no erosion. 17,000 years.
Too many unknowns -- not enough equations

In general, however, we know a significant amount of erosion has taken place.

Also, many landforms have cosmogenic-nuclide inheritance -- nuclide inventory developed elsewhere before deposition.

Thus, in many situations there are not two but three unknowns.

\[ N = N_{inh} + \frac{P}{\lambda + \epsilon/\Lambda} \left[ 1 - e^{-t_{exp}(\lambda + \epsilon/\Lambda)} \right] \]
Depth profiles accurately determine inheritance

Obvious solution: collect more measurements that are linked in some way. Try to obtain a well-posed system that permits solving for the age, erosion rate, and inherited Be-10 concentration.

One method (which I won’t talk about) involves measuring two nuclides with different half-lives in the same surface. Not useful in most situations.

A second method is the depth profile method. Assumes constant inheritance; asymptotically decreasing postdepositional production. This allows accurate determination of the inheritance.
But don’t make progress on $E$ vs. $t$.

Example:

Inheritance and new production accurately separated.

However, any combination of age and erosion rate will equivalently yield an exponential profile in the shallow region. Thus, we have not made progress on uniquely determining $t$ and $E$.

Example:
Inherited Be-10: 1.1 Matoms/g
Postdepositional: 7.1 Matoms/g

Apparent $t$: 0.4 Ma
Apparent $E$: 2 m/Myr
Deep depth profiles -- two production pathways

Deep (several meters) depth profiles sample two production pathways: spallation and muon interactions.

Different pathways have approximately exponential depth dependences — but different attenuation lengths.
Deep depth profiles -- theory

Deep (several meters) depth profiles sample two production pathways: spallation and muon interactions.

Different pathways have approximately exponential depth dependences -- but different attenuation lengths.

\[ N = \frac{P}{\lambda + \frac{\epsilon}{\Lambda}} \left[ 1 - e^{-t_{\text{exp}}(\lambda + \epsilon/\Lambda)} \right] \]

Thus, samples where total Be-10 reflects different production processes have different \( E \) vs. \( t \) solution sets.

This theoretically permits solution for both \( E \) and \( t \).
Deep depth profiles - reality

This seems awesome, but there are some serious problems.

In the muon-dominated zone, inherited Be-10 is usually much larger than muon-produced Be-10. (Big number - big number) = small number.

Second, production rates by muons are critical to this exercise -- and we don’t know them very well.

Third, this procedure depends critically on the assumption of steady erosion. If erosion rates change, then the model framework is invalid and we get the wrong answer.

As precise a depth profile as we can expect gives only weak leverage on exposure time vs. erosion rate.

And this doesn’t even include production rate uncertainty.
Different method: geologically-based exposure model

The rest of this talk will be about a different method.

Be-10 measurements from a variety of surfaces on the same landform (an alluvial fan)

The exposure histories of these surfaces are different, but linked by common geomorphic processes

I will put together an “exposure model” that:

Has input parameters of exposure age, erosion rate, and nuclide inheritance

Is based on simple geomorphology

Predicts observed Be-10 concentrations

And use it to choose the age and erosion rate that fit all the data together.
Goal: age that the fan was emplaced. This constrains slip rate of San Andreas.

Three different studies attempted to date this with Be-10 measurements. Came up with incompatible conclusions.

I'm going to show that a simple geomorphic model with three free parameters reconciles these.
Study of van der Woerd et al. (JGR 2006)

- Exposure-dated cobbles from fan surface
- Asserted that there was neither inheritance nor erosion, so the mean of cobbles ages (35 ka) gives the age of the fan

From van der Woerd et al., JGR 06
Study of Behr et al. (GSAB 2009)

- Exposure-dated boulder tops
- Interpreted height-apparent age relation to indicate erosion
- Concluded that older boulders (45-50 ka) best approximated fan age
Inherited Be-10 is a large fraction of the total Be-10.

This means:

Assertions of other studies that no inheritance is present in clasts or boulders are probably wrong.

Apparent surface age when inheritance is accounted for is ca. 30 ka. However, the depth profile accurately constrains inheritance, but not erosion rate vs. exposure age.
All data from Biskra Palms fan

Wide range of apparent exposure ages from ca. 20 ka to 55 ka.

Overall elevation-nuclide concentration relationship.

Large scatter in above-ground data.
Exposure model -- basic rules

1. Allow for the possibility that the fan surface has eroded.

2. Boulder tops are fixed in place. Thus, boulders that are taller than the total amount of erosion have been exposed since fan emplacement; those that are shorter were originally covered.

3. Surface cobbles aren’t fixed in place, so can be lowered by deflation. Some may have been at the surface when the fan was emplaced, and let down by deflation. Others may have been exposed very recently.

4. The depth profile has constant inheritance and no vertical mixing.

5. Allow for the possibility that all boulders and cobbles contain some inherited Be-10. The maximum and minimum possible amount of inherited Be-10 in boulders and cobbles is related to the inherited Be-10 in fine sediment (that we see in the depth profile) by the fact that both boulders and fine sediment come from the same place.

[6. There is a small amount of surface erosion of boulder and cobble surfaces (this turns out to be unimportant). ]
Exposure model -- linking sediment and boulder inheritance

Assume: boulders, cobbles, and fine sediment that forms the depth profile are all detached from the same surfaces in the watershed.

Fine sediment has the surface concentration.

Boulders must have concentration somewhere between the surface concentration and the concentration at the bottom of the boulder.

Thus, a larger possible range of inheritance is possible for larger boulders.

We never know which side of the boulder we sampled after it was deposited on the fan, so we can only say that the inheritance must be within this range.

In a steadily eroding surface, the Be-10 concentration decreases exponentially with depth.
Exposure model -- specifies time-depth histories

These assumptions specify the time-depth history of all samples: boulder tops, surface cobbles, and depth profile samples, as a function of two unknown parameters: the total erosion depth $E$ (or equivalently an erosion rate) and the exposure time $t$.

$$N_i = \int_0^t P(z_i(T)) e^{-\lambda T} dt$$

With these time-depth histories, we can specify the nuclide concentrations in all samples as a function of three free parameters: depth profile inheritance, erosion depth, and exposure time.
Exposure model - prediction

Three input parameters: age, total amount of erosion, and inheritance.

Predicts all the observables:

- Be-10 depth profile
- Permitted range of ages for surface cobbles
- Permitted range of ages for boulder tops at same height
- Height-age relationship for boulder tops

Model prediction for 45 ka exposure, 1 m surface erosion

- Height-age relationship and range of permissible ages for boulder tops
- Depth profile
- Range of permissible ages for surface cobbles
- Apparent exposure age at surface production rate (yr)
**Exposure model fit to data**

Model does a good job of fitting data. All seemingly disparate cosmogenic-nuclide measurements explained by a simple geomorphic model.

Has a best-fit solution. However, a large range of exposure times and erosion rates yield an acceptable fit to the data.

<table>
<thead>
<tr>
<th>Exposure time (yr)</th>
<th>Total surface erosion (m)</th>
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<tr>
<td>3</td>
<td>3.5</td>
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<tr>
<td>4</td>
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<tr>
<td>7</td>
<td>5.5</td>
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</tbody>
</table>

Apparent exposure age at surface production rate (yr)

Continuous-erosion model - best fit - 69.3 ka, 3.2 m

Range of permissible ages for surface cobbles

Best fit

Acceptable fit
Exposure model fit to data

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Additional constraints from independent observations

One can also use independent constraints on the unknown parameters to limit which of the age-erosion pairs permitted by the cosmogenic-nuclide measurements are actually possible.
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Examples:

Soil stratigraphy (depth of Bk horizon) suggests that surface erosion has been no more than 1.5 or at most 2 m.
Additional constraints from independent observations

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Examples:

Soil stratigraphy (depth of Bk horizon) suggests that surface erosion has been no more than 1.5 or at most 2 m.

U-Th ages on soil carbonate give a minimum fan age of 45 ka.

Only ages 45-50 ka and 1.5-2 m erosion fit all constraints.
Different geologic story -- different results

Remember we specified that erosion was steady and continuous during entire period of exposure. What if this wasn’t true?

For example, suppose that all the erosion occurred rapidly during the last termination near 20-15 ka, as suggested by many regional observations.
Different geologic story -- different results

This yields a slightly different prediction for the boulder ages, and correspondingly a different best-fit solution. This model does a slightly better job and there is a clear best fit at ca. 45 ka, 1 m erosion.

![Graph showing exposure time vs. surface erosion depth with best fit at 41 ka, 1.1 m and range of permissible ages for surface cobbles.](image-url)
**Different geologic story -- different results**

In slightly better agreement with constraints on the erosion rate from soil stratigraphy.

This highlights the importance of understanding the geologic story FIRST. Several geologic scenarios are compatible with all the observations. However, they imply different fan ages and hence slip rates on the San Andreas Fault.
Conclusions

A very simple model shows that all these disparate data sets are consistent with each other.

However, even taken together we cannot use the cosmogenic-nuclide data to uniquely infer the fan age.

With additional constraints from other observations, we do better.

This highlights the need to understand the geological situation before collecting exposure-age data. Use the geology and geomorphology to determine WHAT happened. Then use a forward model like this one to design a cosmogenic-nuclide sampling program to determine WHEN it happened.